

# THE THINKING GUIDE:

## *A Depth & Complexity Student Workbook*



# MATH

*Students are immersed in Deep & Complex mathematical processes and procedures!*

DISCOURSE ..... APPLICATION

INQUIRY



RELATIONSHIPS

PROCEDURES



CONCEPTUAL UNDERSTANDING



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# MATH

The Workbook is a useful tool to support classroom learning. Each page of the workbook incorporates elements of the Depth and Complexity framework in order to facilitate authentic engagement, critical thinking, and open-endedness for students.

The primary objective of the Depth and Complexity framework is to provide students with challenging learning opportunities by applying thinking processes to the study of the core curriculum. The icons of Depth and Complexity and Content Imperative Icons are prompts used to initiate a more advanced and extensive thinking process. Students will be engaged and appropriately challenged as they work through the activities and formulate understanding of subject matter.

The differentiated activities in this workbook are open-ended making the content accessible and appropriate for all levels of learners. All activities blend iconic prompts and critical thinking with grade level content.

This workbook contains 6 differentiated activities for each element within a highly effective math lesson as well as blank note pages. It is recommended that students work through each page, completing all of the activities, over the course of a unit. However, this workbook does not need to be completed in any order; the classroom teacher may select activities to support learning of content.



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# Thinking Tools: The Icons of Depth and Complexity and the Content Imperatives

Name	Icon	Definition	“Verb” It is the process of...
Big Idea		The generalization, principles, and theories that distinguish themselves from the facts and concepts of the area or discipline under study.	Gathering all the information and details to generalize the main idea.
Details		Specific characteristics that describe a concept, theory, or even a fact.	Identifying the important items in an area of study.
Unanswered Questions		The ambiguities and gaps of information recognized within an area or discipline under study.	Determining what is still unknown or ambiguous while providing room to make a prediction in an area of study.
Patterns		Recurring events represented by details; sequences of order of events.	Examining commonalities that repeat or are easily predictable.
Trends		Refers to factors that influence events; political, social, economic, or religious factors that influence / create a pattern.	Analyzing commonalities of human influence that repeat or are easily predictable in an area of study.
Ethics		The controversial issues that plague an area of study; moral values involved; biases that exist.	Judging the degree of right or wrong, good or bad; all depend on the perspective.
Rules		The natural or person-made structure or order of things that explains the subject in study.	Looking for structure and / or order in an area of study.
Language of the Discipline		The specific specialized and technological terms associated with a specific area of study.	Analyzing, identifying, and interpreting the content-specific vocabulary.
Over Time		The understanding of time as an agent of change and recognition that the passage of time changes our knowledge of things.	Examining how time affects or changes a concept or area of study.
Across Disciplines		Connections made within, between, and among various areas of study or disciplines.	Examining one curricular area and then finding similarities and connections to other curricular areas.
Multiple Perspectives		The concept that different points of view alter the way ideas and objects are viewed and valued.	Examining or analyzing the different points of view (including thoughts, emotions, values, motives, etc.) in an area of study.
Origin		The beginnings, roots, foundations, or causes.	Understanding, determining, and exploring the emergence of a topic.
Contribution		The lasting effects; the results or product; the influence of a topic, event, or person.	Determining the effects or results of a particular event, action, and / or aspect.
Convergence		Events, items that merged or happened concurrently in order for a particular result to occur.	Examining all of the aspects, details, and factors that come together in order for something to occur or exist.
Parallel		The similarities, events, people, problems, principles, that compare or correspond; compare or correspond events, people, problems, and principles.	Analyzing the commonalities and differences between two or more aspects.
Paradox		Contradictions regarding opinion, statement, events, situation, or area of study; opposites.	Examining the simultaneous duality between two concepts or two aspects.

# DISCOURSE

The “DISCOURSE” activities provide multiple opportunities for the verbal exchange of representing, communicating, and thinking about the ways student perform mathematical tasks. These activities support the development of conceptual understanding.

## Example Activities:

The “Ponder” activity provides students space to do a “brain dump” to detail all of their prior knowledge. You may choose the icon for focus. Student write and/or illustrate what they already know about the topic.

### Workbook

**DISCOURSE: Ponder** Date: \_\_\_\_\_

Directions: What do you already know about this topic? What connections come to mind? What do you wonder? Draw an icon in the box. Use the icon to guide and record your thinking. Discuss your thoughts with a partner.

TOPIC: \_\_\_\_\_

PONDER  ICON




### Student Sample Work

**DISCOURSE: Ponder** Date: \_\_\_\_\_

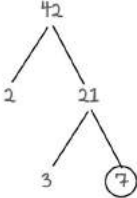
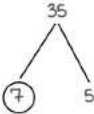
Directions: What do you already know about this topic? What connections come to mind? What do you wonder? Draw an icon in the box. Use the icon to guide and record your thinking. Discuss your thoughts with a partner.

TOPIC: Finding the greatest common factor

PONDER  ICON

I know that the greatest common factor is always going to follow the rule that it is the largest number that both numbers are divisible by. Solve the greatest common factor I know that you need to follow the steps and list out all of the numbers that a number is divisible by.

Example



The rule that you have to follow is find the biggest number that both numbers have in common and that is how you can find the greatest common factor.



# INQUIRY

The “*INQUIRY!*” activities provide multiple opportunities for students to explore a mathematical prompt or concept. Students ask questions, make conjectures and generalizations, share information, and collaborate with others.

## Example Activities:

The “Details” activity allows students to explore the different ways to find a number or solution in order to build number sense.

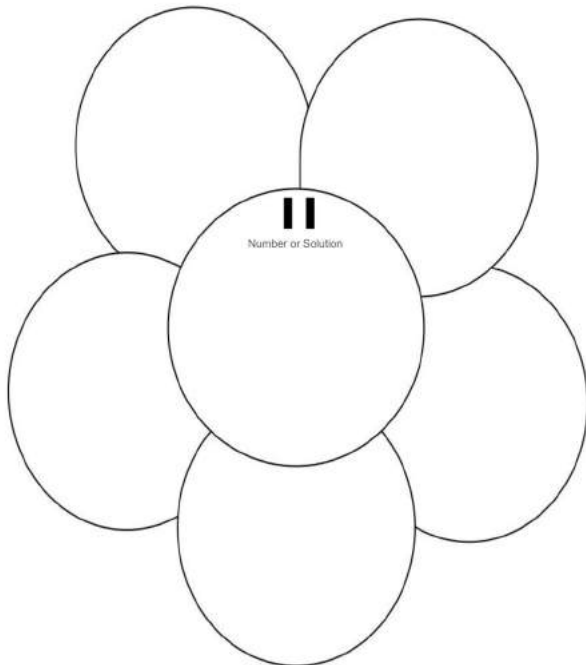
### Workbook

*INQUIRY: Blooming Parallel*

Date: \_\_\_\_\_

Directions: Write a number or solution in the center. In each petal of the detail icon (flower) write a different way to find the number or solution.

TOPIC: \_\_\_\_\_



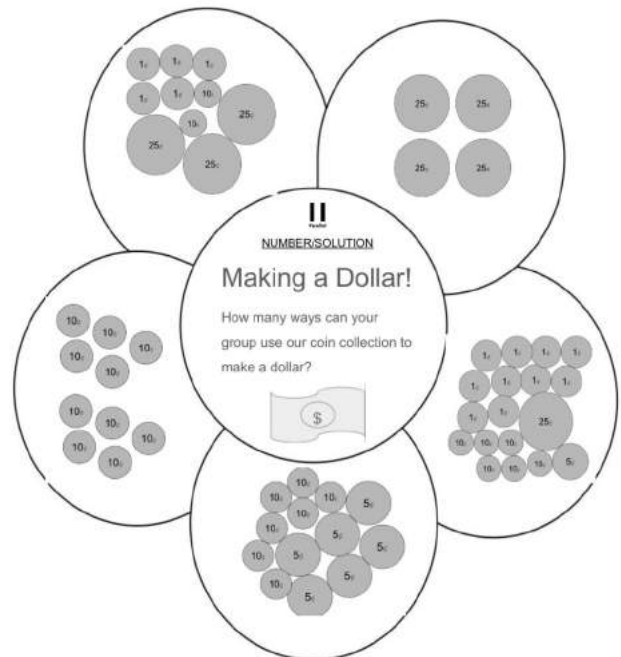
### Student Sample Work

*INQUIRY: Blooming Parallel*

Date: \_\_\_\_\_

Directions: Write a number or solution in the center. In each petal of the detail icon (flower) write a different way to find the number or solution.

TOPIC: Making a dollar



# RELATIONSHIPS

The “*RELATIONSHIPS!*” activities provide multiple opportunities for students to make connections between numbers or variables.

## Example Activities:

The “*RELATIONSHIPS: Perspective Pathway*” activity allows students to reflect on how they think about this math concept.






### Workbook

### Student Sample Work

*RELATIONSHIPS: Perspective Pathway* Date: \_\_\_\_\_

Directions: As you learn this math concept, use the icons and prompts to guide your thinking and organize your notes.






TOPIC: \_\_\_\_\_

 Practice/Examples:	 What would be confusing or need to be explained if this was the first time learning this? List and explain.	 Sequence the steps:
	 It's easy if...	 Where could someone go wrong if they don't follow the rules?

*RELATIONSHIPS: Perspective Pathway* Date: \_\_\_\_\_

Directions: As you learn this math concept, use the icons and prompts to guide your thinking and organize your notes.

TOPIC: Multiplying Decimals

 Practice/Examples:	 What would be confusing or need to be explained if this was the first time learning this? List and explain.	 Sequence the steps:
	 It's easy if...	 Where could someone go wrong if they don't follow the rules?





# PROCEDURES



The “*PROCEDURES!*” activities provide multiple opportunities for students to sequence mathematical operations in order to solve an equation.

## Example Activities:






The “*PROCEDURES: Solution Finder*” activity allows students to break down the vocabulary, essential information, and important steps before they solve a problem.

### Workbook

*PROCEDURES: Solution Finder*      Date: \_\_\_\_\_



**Directions:** As you learn this math procedure, use the icons and prompts to guide your thinking and organize your notes. The   is where you solve the problem, showing all of your steps.

TOPIC: \_\_\_\_\_









Problem		
 <b>Key Words:</b>	 <b>Essential Information:</b>	 <b>Important Rules:</b>
  <b>Solution:</b>		

### Student Sample Work

*PROCEDURES: Solution Finder*      Date: \_\_\_\_\_

**Directions:** As you learn this math procedure, use the icons and prompts to guide your thinking and organize your notes. The   is where you solve the problem, showing all of your steps.

TOPIC: \_\_\_\_\_

Problem I will be able to identify prime and composite numbers.		
 <b>Key Words:</b> <u>Prime Numbers:</u> Prime: A whole number whose the factors are only one and itself. Factor: Whole numbers that divide another without leaving any remainder.	 <b>Essential Information:</b> Examples: 2, 3, 7, 13, 29 <div style="text-align: center;"> <math display="block">\begin{array}{c} 2 \\ \swarrow \quad \searrow \\ 1 \quad 2 \end{array}</math> <math display="block">\begin{array}{c} 5 \\ \swarrow \quad \searrow \\ 1 \quad 5 \end{array}</math> <math display="block">\begin{array}{c} 37 \\ \swarrow \quad \searrow \\ 1 \quad 37 \end{array}</math> </div>	 <b>Important Rules:</b> <u>Prime Numbers:</u> 1. Find the factors for the number. 2. If the number is prime it will be <u>only</u> divisible by 1 and itself.
 <b>Key Words:</b> <u>Composite Numbers:</u> Composite: Numbers that have two or more factors. Factor: Whole numbers that divide another without leaving any remainder.	 <b>Essential Information:</b> Examples: 4, 8, 12 <div style="text-align: center;"> <math display="block">\begin{array}{c} 4 \\ \swarrow \quad \searrow \\ 2 \quad 2 \end{array}</math> <math display="block">\begin{array}{c} 8 \\ \swarrow \quad \searrow \\ 2 \quad 4 \\ \swarrow \quad \searrow \\ 2 \quad 2 \end{array}</math> <math display="block">\begin{array}{c} 12 \\ \swarrow \quad \searrow \\ 3 \quad 4 \\ \swarrow \quad \searrow \\ 2 \quad 2 \end{array}</math> </div>	 <b>Important Rules:</b> <u>Composite Numbers:</u> 1. Find the factors for the number. 2. If the number is composite it will be divisible by <u>more</u> than one and itself.
  <b>Solution: Factor and Label Numbers As Prime or Composite.</b> 3: 1, 3- Prime 7: 1, 7- Prime 6: 1, 2, 3, 6 -Composite 36: 1, 2, 3, 4, 6, 9, 12, 18, 36 -Composite		

# APPLICATION

The “APPLICATION!” activities provide multiple opportunities for students to clarify, solidify, and extend mathematical skills and concepts in familiar and new contexts.

## Example Activities:

This activity allows students to explore connections between the mathematical concepts and real world applications.

### Workbook

**APPLICATION: Math Across Disciplines** Date: \_\_\_\_\_

Directions: Where do you see this math concept? What connections come to mind? Fill in the space with your illustrations.

TOPIC: \_\_\_\_\_



### Student Sample Work

**APPLICATION: Math Across Disciplines** Date: \_\_\_\_\_

Directions: Where do you see this math concept? What connections come to mind? Fill in the space with your illustrations.

TOPIC: Shapes in the Classroom





# EXIT TICKET

The “EXIT TICKET!” activities provide multiple opportunities for students to measure their own understanding, give and receive feedback, and present information.

## Example Activities:





The “Exit Ticket” activity provides students with an opportunity to measure their own understanding of the topic. This also may be used as a summative or formative assessment for the teacher.

### Workbook

**EXIT TICKET: Frame It**      Date: \_\_\_\_\_

Directions: Write the problem in the middle box. Use the icons to guide your thinking and examine the math problem. Record your thoughts in each box.

TOPIC: \_\_\_\_\_


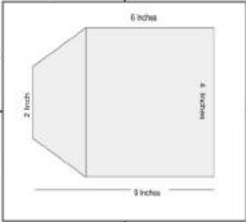



	<p>Problem:</p>	
		

### Student Sample Work

**EXIT TICKET: Frame It**      Date: \_\_\_\_\_

Directions: Write the problem in the middle box. Use the icons to guide your thinking and examine the math problem. Record your thoughts in each box.

TOPIC: Area of composite figures

<p> <b>Details</b></p> <p>Important details to breakdown this problem:</p> <ul style="list-style-type: none"> <li>-Shapes given: Trapezoid rectangles.</li> <li>-The rectangle is 6 inches in length and 4 inches in width.</li> <li>-The trapezoid side a is 2 inches long. The side b is 4 inches long. The height of the trapezoid is found by subtracting 9 from 6 so the height is 3 inches.</li> <li>-Area is the entire shape.</li> </ul>		<p> <b>Patterns</b></p> <ol style="list-style-type: none"> <li>1. Break the shape into common shapes.             <ul style="list-style-type: none"> <li>-Trapezoid and a rectangle.</li> </ul> </li> <li>2. Find the formulas needed to find the area of the common shapes.             <ul style="list-style-type: none"> <li>-Rectangle <math>A = L \times W</math></li> <li>-Trapezoid <math>A = a + b \div 2 \times h</math></li> </ul> </li> <li>3. Find the area of each shape.</li> <li>4. Add the total area of the shapes.</li> </ol>
<p> <b>Unanswered Questions</b></p> <p>-The problem is to find the area of the entire shape.</p> <p>-What is the best way to breakdown this shape?</p> <p>-What common shapes make up this shape?</p> <p>-What are the formulas for the common shapes?</p> <p>-If I wanted to find using areas of a triangles instead of trapezoid is that possible?</p>		<p> <b>Rules</b></p> <p><math>A = L \times W</math></p> <p><math>6 \times 4 = 24 \text{ in}^2</math></p> <p><math>A = \frac{a+b}{2} \times h</math></p> <p><math>\frac{2+4}{2} \times 3</math></p> <p><math>A = 9 \text{ in}^2 + 24 \text{ in}^2</math></p> <p><math>A = 33 \text{ in}^2</math></p> <p><math>\frac{6}{2} \times 3 = 9 \text{ in}^2</math></p>